Reflection and Transmission Coefficients of Rectangular Dielectric Waveguide Discontinuity with an Air Gap

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***Abstract*—Reflection, transmission, and radiation characteristics of rectangular dielectric waveguide discontinuities with an air gap are analyzed. A case where the guide axes are displaced in transverse directions is also treated. The characteristic of this discontinuity has never been analyzed by a conventional slab model. In this analysis, a two-dimensional Fourier transformation and orthogonal relations between guided and radiation modes are utilized. Numerical calculations and experimental results in X-band are compared, and good agreement is found. To discuss our numerical results, the principle of energy conservation is introduced. It becomes evident that the discrepancy between our results and this principle is less than 0.0003.**

I. Introduction

The dielectric waveguide with a rectangular cross section (RDWG) plays an important role in integrated optics. It is necessary to analyze a characteristic at abrupt discontinuities on the guide such as a truncated end or a noncoincidental junction. In conventional approaches to this problem, a slight discontinuity or a simple two-dimensional slab structure model was treated [1], [2]. Using the Fourier transformation, Takenaka *et al.* [3] analyzed two-dimensional discontinuity problems. We have analyzed reflection, transmission, and radiation characteristics at three-dimensional discontinuities such as the truncated end of the guide [4], the coaxial junction between guides of different sizes [5], and the noncoincidental junction of guides with the same dimension [6].

Recently, numerical analyses of a symmetric double step on a slab waveguide with an air gap [7] and of misalignment with an air gap between an optical fiber and a slab waveguide have been reported [8].

In this paper, we analyze the reflection, transmission, and radiation characteristics where two RDWG ends are facing each other with an air gap. A transverse displacement with the air gap is also treated. The characteristics of this discontinuity have never been analyzed by the slab structure model. Analytical results of the transmission coefficient are in good agreement with those of X-band experiments.

II. Analysis

A discontinuity with an air gap between RDWG #1 and #2 and a coordinate system are shown in Fig. 1. RDWG #1 and #2 are assumed to be single-mode waveguides having the same dimensions. Truncated faces of RDWG #1 and #2 are located at $z=0$ and $z=L$, respectively, and they are perpendicular to the propagation axes. The displacements of the propagation axes of the two waveguides along the x and y axes are denoted by c and d, respectively. Both guides have the same dielectric constant in the core ϵ_r and in the cladding ϵ_c.

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A fundamental E_y^{11} mode [9] propagates along RDWG #1 from negative infinity of z. Because transverse components of the E_y^{11} mode are mainly E_y and H_x, the boundary condition equations on the discontinuities can be derived as follows. On the discontinuity at $z=0$,

$$\begin{aligned} &E_y^i(x,y)+R_0E_y^i(x,y) \\ &\quad+\int_0^\infty\int_0^\infty R^{\mathrm{I}}(\rho_x,\rho_y)E_y^{t\mathrm{I}}(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \\ &=\int_0^\infty\int_0^\infty T^{\mathrm{I}}(\rho_x,\rho_y)E_y^{t\mathrm{I}}(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \\ &\quad+\int_0^\infty\int_0^\infty R^{\mathrm{II}}(\rho_x,\rho_y)E_y^{t\mathrm{II}}(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \end{aligned} \tag{1}$$

$$\begin{aligned} &H_x^i(x,y)-R_0H_x^i(x,y) \\ &\quad-\int_0^\infty\int_0^\infty R^{\mathrm{I}}(\rho_x,\rho_y)H_x^{t\mathrm{I}}(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \\ &=\int_0^\infty\int_0^\infty T^{\mathrm{I}}(\rho_x,\rho_y)H_x^{t\mathrm{I}}(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \\ &\quad-\int_0^\infty\int_0^\infty R^{\mathrm{II}}(\rho_x,\rho_y)H_x^{t\mathrm{II}}(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y. \end{aligned} \tag{2}$$

On the discontinuity at $z=L$,

$$\begin{aligned} &\int_0^\infty\int_0^\infty T^{\mathrm{I}}(\rho_x,\rho_y)E_y^{t\mathrm{I}}(\rho_x,\rho_y,x,y)\exp\{-j\beta_t(\rho_x,\rho_y)L\}\,d\rho_x\,d\rho_y \\ &\quad+\int_0^\infty\int_0^\infty R^{\mathrm{II}}(\rho_x,\rho_y)E_y^{t\mathrm{II}}(\rho_x,\rho_y,x,y) \\ &\quad\cdot\exp\{j\beta_r(\rho_x,\rho_y)L\}\,d\rho_x d\rho_y \\ &\quad=T_0E_y^t(x,y)\exp(-j\beta_gL) \\ &\quad+\int_0^\infty\int_0^\infty T^{\mathrm{II}}(\rho_x,\rho_y)E_y^{t\mathrm{II}}(\rho_x,\rho_y,x,y) \\ &\quad\cdot\exp\{-j\beta_t(\rho_x,\rho_y)L\}\,d\rho_x\,d\rho_y. \end{aligned} \tag{3}$$

$$\begin{aligned} &\int_0^\infty\int_0^\infty T^{\mathrm{I}}(\rho_x,\rho_y)H_x^{t\mathrm{I}}(\rho_x,\rho_y,x,y)\exp\{-j\beta_t(\rho_x,\rho_y)L\}\,d\rho_x\,d\rho_y \\ &\quad-\int_0^\infty\int_0^\infty R^{\mathrm{II}}(\rho_x,\rho_y)H_x^{t\mathrm{II}}(\rho_x,\rho_y,x,y) \\ &\quad\cdot\exp\{j\beta_r(\rho_x,\rho_y)L\}\,d\rho_x\,d\rho_y \\ &\quad=T_0H_x^t(x,y)\exp(-j\beta_gL) \\ &\quad+\int_0^\infty\int_0^\infty T^{\mathrm{II}}(\rho_x,\rho_y)H_x^{t\mathrm{II}}(\rho_x,\rho_y,x,y) \\ &\quad\cdot\exp\{-j\beta_t(\rho_x,\rho_y)L\}\,d\rho_x\,d\rho_y. \end{aligned} \tag{4}$$

In these expressions, R_0 and T_0 are the reflection and transmission coefficients of the guided mode. The electric and magnetic fields of the guided mode are written $E_y(x,y)$ and $H_x(x,y)$. Terms expressed by integral forms are radiation modes generated at the discontinuities. $R(\rho_x,\rho_y)$ and $T(\rho_x,\rho_y)$ are the reflection and transmission coefficients of the radiation mode. The superscripts i, r, and t indicate the incident, reflection, and transmission components. In the integrals, the superscripts I and II refer to RDWG #1 and #2, respectively.

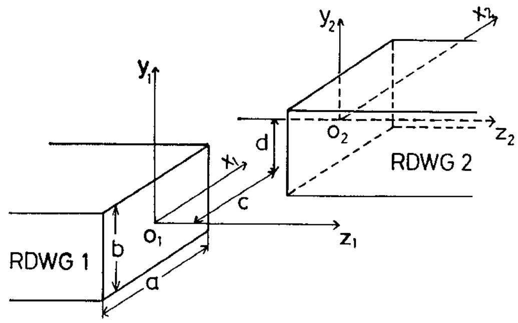

Fig. 1. Rectangular dielectric waveguide discontinuity with an air gap and coordinate system.

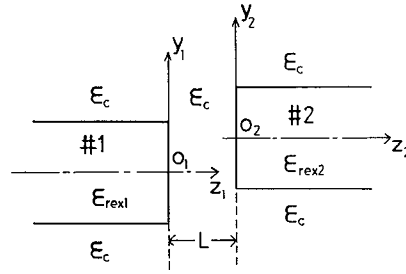

Fig. 2. Dielectric constant distributions near the discontinuity.

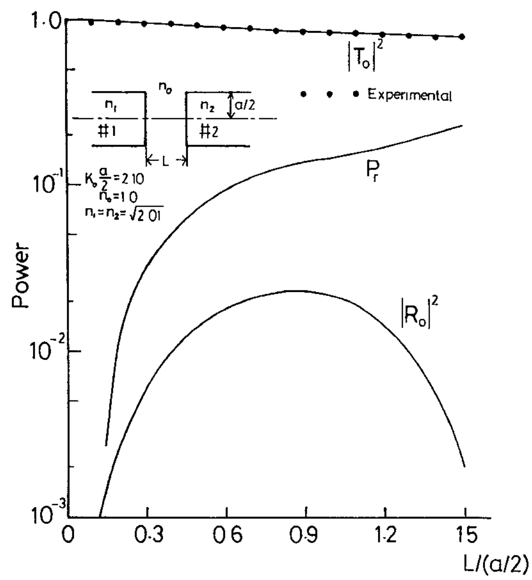

Fig. 3. Reflection, transmission, and radiation powers versus normalized air gap width.

Utilizing the two-dimensional Fourier transformation, we can express the boundary condition equation as follows:

$$\begin{bmatrix} \dfrac{p_1(\mu,\nu)}{\epsilon_+} & q(\mu,\nu) & -q(\mu,\nu) & 0 \\ 0 & -q(\mu,\nu)\exp\{-jq(\mu,\nu)L\} & q(\mu,\nu)\exp\{jq(\mu,\nu)L\} & \dfrac{p_2(\mu,\nu)}{\epsilon_-}\exp\{-jp_2(\mu,\nu)L\} \\ 1 & -1 & -1 & 0 \\ 0 & \exp\{-jq(\mu,\nu)L\} & \exp\{jq(\mu,\nu)L\} & -\exp\{-jp_2(\mu,\nu)L\} \end{bmatrix} \begin{bmatrix} \mathscr{H}^{r\mathrm{I}} \\ \mathscr{H}^{t\mathrm{I}} \\ \mathscr{H}^{r\mathrm{II}} \\ \mathscr{H}^{t\mathrm{II}} \end{bmatrix}$$

$$= \begin{bmatrix} (1+R_0)\dfrac{\beta_g}{\epsilon_+}h^t(\mu,\nu) \\ -T_0\left\{\dfrac{\beta_g}{\epsilon_-}h^t(\mu,\nu)\exp(-j\beta_g L)\right\} \\ (R_0-1)h^t(\mu,\nu) \\ T_0 h^t(\mu,\nu)\exp(-j\beta_g L) \end{bmatrix}. \quad (5)$$

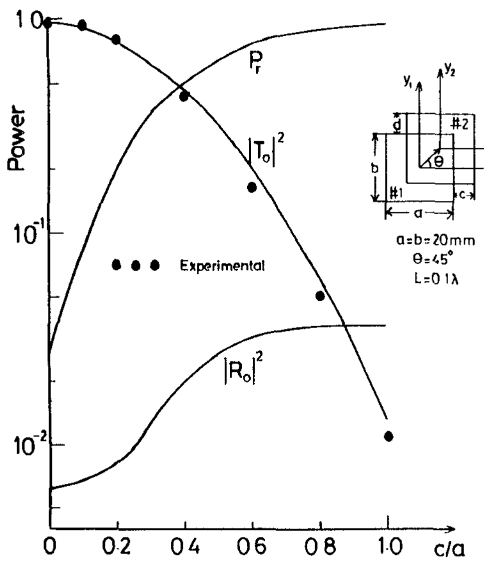

Fig. 4. Reflection, transmission, and radiation powers versus normalized transverse displacement.

In this equation, the electric fields in the spectral domain are expressed by means of the magnetic fields. The $h(\mu,\nu)$ and $\mathscr{H}(\mu,\nu)$ are spectral expressions of the magnetic field of the guided and the radiation mode, respectively. β_g is the phase constant of the guided mode. The parameters $p_1(\mu,\nu)$, $q(\mu,\nu)$, and $p_2(\mu,\nu)$ correspond to the phase constants of the radiation mode in three regions ($z<0$, $0<z<L$, $z>L$) divided by the discontinuities at $z=0$ and $z=L$. The ϵ_- and ϵ_+ show the dielectric constant distributions in the region of $z<0$ and $z>L$, and they are given in Fig. 2. In this figure ϵ_{rex1} and ϵ_{rex2} are effective dielectric constants obtained by the generalized effective dielectric constant method [10].

A method to derive R_0 and T_0 is omitted here, because it is very similar to the procedure given in [6, sec. II-B].

III. Numerical Results and Experimental Investigation

A. Coaxial Junction with an Air Gap

It is necessary to know the transverse field expression of the guided modes to carry out the procedure described above. In this paper, an approximate guided field expression [6] is used. In an example of numerical calculations, RDWG's #1 and #2, whose cross sections are 20 mm by 20 mm, are coaxially arranged with the air gap. The dielectric constants of the core and cladding regions are 2.01 and 1.0, respectively. The fundamental mode E_y^{11} with an amplitude corresponding to 1 [W] is excited at negative infinity in the z direction. The frequency is 10 GHz.

Variations of $|R_0|^2$, $|T_0|^2$, and radiation power P_r with the normalized air gap $L/(a/2)$ are presented in Fig. 3. In this figure, P_r is a sum of forward and backward radiation powers. The difference between unity and the sum of these powers shows a discrepancy from the principle of energy conservation. In this result, the difference is less than 0.0003.

The experimental results in X-band of the transmission coefficient of the guided mode are also shown by dotted marks in this figure. It becomes evident that the numerical result gives good agreement with the experimental one.

B. Noncoincidental Junction with an Air Gap

The characteristics of this discontinuity have never been analyzed with the two-dimensional slab model. The propagation axis of RDWG #2 is displaced by c and d along the x and y axes, respectively. The guided field expressions around RDWG #2 in the spectral domain are presented in [6]. The dimensions of the RDWG and the calculation procedures are the same as in subsection III-A.

In Fig. 4, the variation of $|R_0|^2$, $|T_0|^2$, and the radiation power P_r with the normalized transverse displacement c/a is presented for the air gap $L=0.1\lambda$ (λ the wavelength in free space). In this figure, the experimental transmission coefficients (dotted marks) are in good agreement with the numerical one. The discrepancy from the principle of energy conservation in this case is also smaller than 0.0003.

IV. Conclusion

The reflection, transmission, and radiation characteristics on the RDWG discontinuity where the guide ends are facing each other with an air gap are analyzed with the Fourier transformation method. The characteristics for the case where the propagation axis is displaced in the x and y directions are also presented. These characteristics have never been obtained by the conventional slab approximation. It is found that the discrepancy from the principle of energy conservation is less than 0.0003. The experimental result in X-band of the transmission coefficient of the guided mode shows good agreement with the analytical one.

The analytical method and the results of this study will be useful in giving a criterion of the discontinuities in an optical integrated circuit.

References

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